

# Black hole evaporation based upon a $q$ -deformation description

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## Abstract

A toy model based upon the  $q$ -deformation description for studying the radiation spectrum of black hole is proposed. The starting point is to make an attempt to consider the spacetime noncommutativity in the vicinity of black hole horizon. We use a trick that all the spacetime noncommutative effects are ascribed to the modification of the behavior of the radiation field of black hole and a kind of  $q$ -deformed degrees of freedom are postulated to mimic the radiation particles that live on the noncommutative spacetime, meanwhile the background metric is preserved as usual. We calculate the radiation spectrum of Schwarzschild black hole in this framework. The new distribution deviates from the standard thermal spectrum evidently. The result indicates that some correlation effect will be introduced to the system if the noncommutativity is taken into account. In addition, an infrared cut-off of the spectrum is the prediction of the model.

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It is well known that the noncommutative geometry [1] plays a very important role in revealing the properties of Planck scale physics [2–4]. It has been suspected for a long time that the noncommutative spacetime might be a realistic picture of how spacetime behaves near the Planck scale. The most realistic laboratory for testing Planck scale physics is the early universe and the black hole. In the studies of the early universe, it has been shown [5] that if the spacetime is indeed noncommutative on short distance, this may have an impact on early universe such as the evolution of the primordial fluctuation spectrum generated during the inflation (for spacetime noncommutative inflation, see also [6]). Moreover, it can be imagined naturally that the spacetime noncommutative effects in the vicinity of black hole horizon might bring influence to the fundamental physics in some degree. However, the investigation for testing those noncommutative effects near black hole is extremely difficult due to the fact that a consistent theory of quantum gravity is not available at present, let alone a theory of noncommutative quantum gravity. Still, some efforts have been made on this subject [7–9]. It should be mentioned that by considering a semi-classical theory of noncommutative fields, the authors of Ref. [8] give some remarks on 't Hooft's brick wall model [10].

In this note, we will make an attempt to consider the spacetime noncommutative quantum effects near the black hole by using a deformation description of quantum fields. The deformation description of physical systems can be traced back to the quantum group symmetry. Quantum groups lead to an algebraic structure that can be realized on quantum spaces, and such quantum spaces can be interpreted as noncommutative configuration spaces for physical systems [11]. Moreover, it has been demonstrated in Ref. [11] that in the  $q$ -deformed algebraic structure the spacetime occurs in different phases — a lattice phase and a continuous phase. It might be that for very small distances and high energy density the lattice phase is dominant and provides a natural ultraviolet cutoff. Studies on the deformation description for physical systems have made great progresses. The dynamics of a deformed photonic field interacting with matter presented many interesting properties in the area of the quantum optics [12]. Great success has also been achieved by making use of a deformed oscillator to study the molecular spectrum [13]. In addition, the thermodynamics of a deformed oscillator system has been investigated deeply [14]. In particular, it should be pointed out that the  $q$ -deformed noncommutative theory has been presented for resolving the origin of the ultrahigh energy cosmic ray and the TeV-photon paradoxes [15].

The prediction of black hole evaporation [16] is a great triumphant of the combination of general relativity and quantum mechanics, even though this combination is not complete. However, the laws of quantum fields in curved spacetime are only semi-classical such that which should be valid only for cases of relatively low energy scale. As the size of black hole

approaches extremely small distance scale, for instance, the Planck scale or so, the spacetime noncommutativity might play an important role in some courses of fundamental physics, e.g. the black hole evaporation. The consideration of spacetime noncommutativity perhaps helps resolving the problem of information loss [17] (if the radiation spectrum deviates the standard thermal spectrum, it seems that some quantum hair appears). Though we still have no enough knowledge about the underlying theory — the quantum gravity, we can still make an attempt to consider some noncommutative effects in the framework of quantum field theory in curved spacetime. Therefore, we propose a toy model for testing this noncommutative effect in the process of black hole evaporation. The toy model is based upon such an assumption that from an effective point of view the behavior of the spacetime noncommutative effects is reflected completely by the deformation of the radiation field meanwhile the background metric is preserved as usual. It has been found that such a simple assumption will give rise to interesting results. First, the spectrum we obtained deviates from the perfect thermal spectrum evidently, which indicates that some correlation effects are introduced apparently. This maybe helpful for understanding the problem of information loss. Second, the new spectrum takes on some weird behavior, i.e. infrared divergency. It is interesting for us, because that divergency always implies new physics, so we can view that some unknown physical mechanism which we neglect plays the role. On the other hand, however, perhaps our method is not suitable for low frequencies. In any case, an infrared cut-off is necessary, which seems to take on some UV/IR correspondence principle (e.g. see [18]), since the infrared cut-off is the consequence of the ultraviolet modification [11].

In what follows we will introduce the deformation description to the calculation of the radiation spectrum of the Schwarzschild black hole evaporation. For simplicity only the bosonic field is taken into account in this note. The natural quantum of the quantum field theory on noncommutative geometry is the so-called deformed harmonic oscillator. On the topic of deformed oscillator, a lot of papers have been published [19]. Similar to the case of an ordinary quantum field system, the properties of a noncommutative quantum field system can be described by a collection of the deformed harmonic oscillators with the creation and annihilation operators satisfying the following  $q$ -deformed Weyl-Heisenberg algebraic relations

$$a_q a_q^\dagger - q a_q^\dagger a_q = q^{-N}, \quad [N, a_q] = -a_q, \quad [N, a_q^\dagger] = a_q^\dagger. \quad (1)$$

Obviously, the deformed Weyl-Heisenberg algebra  $\{N, a_q, a_q^\dagger\}$  will be reduced to the ordinary one as the deformation-parameter  $q$  approaches 1. The deformed oscillator is related to the simple harmonic oscillator as follows [20]

$$a_q = a\sqrt{\frac{[N]_q}{N}}, \quad a_q^\dagger = \sqrt{\frac{[N]_q}{N}}a^\dagger, \quad N = a^\dagger a, \quad (2)$$

where we have used the notation  $[x]_q = \frac{q^x - q^{-x}}{q - q^{-1}}$ , and the ordinary boson commutation relations are

$$[a, a^\dagger] = 1, \quad [N, a] = -a, \quad [N, a^\dagger] = a^\dagger. \quad (3)$$

The representation of the deformed Weyl-Heisenberg algebra is obtained by constructing the Fock space,

$$|n\rangle_q = \frac{(a_q^\dagger)^n}{\sqrt{[n]_q!}} |0\rangle, \quad (4)$$

which satisfies

$$a_q^\dagger |n\rangle_q = \sqrt{[n+1]_q} |n+1\rangle_q, \quad a_q |n\rangle_q = \sqrt{[n]_q} |n-1\rangle_q, \quad (5)$$

where  $[n]_q! \equiv [n]_q[n-1]_q \cdots [2]_q[1]_q$ . Using the formalism of the boson realization of the deformed algebra, it can be easily demonstrated that the Fock basis of the deformed algebra is the same as the one of the ordinary algebra, namely

$$|n\rangle_q = \frac{(a_q^\dagger)^n}{\sqrt{[n]_q!}} |0\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle = |n\rangle. \quad (6)$$

In the following, Hawking's computation of black hole evaporation will be briefly reviewed, then a radiation spectrum associated with the  $q$ -deformed scalar particles will be derived by using the above ordinary boson realization of the  $q$ -deformed algebra.

Consider, now, the background spacetime of a Schwarzschild black hole appropriate to a collapsing spherical body. For simplicity, we only consider a single massless scalar field  $\phi$  which is coupled to gravity minimally [21]. Near the past null infinity  $\mathcal{I}^-$ , the quantum field  $\phi$  can be expanded as

$$W_{\text{in}}^{-1} \phi(x) W_{\text{in}} = \sum_{\sigma} (a_{\text{in}}(\sigma) \phi_{\text{in}}(\sigma, x) + a_{\text{in}}^\dagger(\sigma) \bar{\phi}_{\text{in}}(\sigma, x)), \quad (7)$$

where the label “in” denotes the incoming process;  $\{\phi_{\text{in}}\}$  is an orthonormal basis of the one-particle Hilbert space  $\mathcal{H}_{\text{in}}$  which is constructed in Minkowski spacetime, and  $\bar{\phi}_{\text{in}}$  represents the complex conjugate function of  $\phi_{\text{in}}$ ;  $a_{\text{in}}^\dagger$  and  $a_{\text{in}}$  are boson creation and annihilation operators corresponding to  $\{\phi_{\text{in}}\}$ ;  $W_{\text{in}}$  denotes the isomorphism map,  $W_{\text{in}} : \mathcal{L} \rightarrow \mathcal{L}_S(\mathcal{H}_{\text{in}})$ , which associates with each state in  $\mathcal{L}$  the Minkowski spacetime state it “looks like” in the past (here  $\mathcal{L}$  denotes a Hilbert space, and  $\mathcal{L}_S(\mathcal{H})$  represents a symmetric Fock space, for the

detail see [21]); and the notation  $\sigma$  is used to distinguish different modes. Near the future null infinity  $\mathcal{I}^+$  and the event horizon of black hole, the field reads

$$W_{\text{out}}^{-1}\phi(x)W_{\text{out}} = \sum_{\sigma}(a_{\text{out}}(\sigma)\phi_{\text{out}}(\sigma, x) + a_{\text{out}}^{\dagger}(\sigma)\bar{\phi}_{\text{out}}(\sigma, x)), \quad (8)$$

where the subindex “out” denotes the outgoing process;  $\{\phi_{\text{out}}\}$  is an orthonormal basis of the one-particle Hilbert space  $\mathcal{H}_{\text{out}}$  that constructed in flat spacetime,  $a_{\text{out}}^{\dagger}$  and  $a_{\text{out}}$  are boson creation and annihilation operators corresponding to  $\{\phi_{\text{out}}\}$ ;  $W_{\text{out}}$  denotes the isomorphism map,  $W_{\text{out}} : \mathcal{L} \rightarrow \mathcal{L}_S(\mathcal{H}_{\text{out}})$ , which associates with each state in  $\mathcal{L}$  the Minkowski spacetime state it “looks like” in the future.

The next step is to extend  $\phi_{\text{out}}$  to the whole background spacetime. Therefore, near the past null infinity  $\mathcal{I}^-$ , the form of  $\phi'_{\text{out}}$  (the prime is used to denote the extension) can be expressed as the linear combination of  $\phi_{\text{in}}$  and  $\bar{\phi}_{\text{in}}$ ,

$$\phi'_{\text{out}}(\sigma, x) = \sum_{\sigma'}(A_{\sigma\sigma'}\phi_{\text{in}}(\sigma', x) + B_{\sigma\sigma'}\bar{\phi}_{\text{in}}(\sigma', x)), \quad (9)$$

where  $A_{\sigma\sigma'}$  and  $B_{\sigma\sigma'}$  are Bogoliubov transformation coefficients, satisfying the following relations

$$AA^{\dagger} - BB^{\dagger} = 1, \quad AB^T - BA^T = 0. \quad (10)$$

Inversion of (9) leads to

$$\phi'_{\text{in}}(\sigma, x) = \sum_{\sigma'}(A_{\sigma\sigma'}^{\dagger}\phi_{\text{out}}(\sigma', x) - B_{\sigma\sigma'}^T\bar{\phi}_{\text{out}}(\sigma', x)). \quad (11)$$

One of the important issues to consider is how the characterization of the states of the field as “in” states compares with their characterization as “out” states. This is given by the  $S$ -matrix,

$$S = W_{\text{out}}^{-1}W_{\text{in}}. \quad (12)$$

Given any “in” state  $\Psi \in \mathcal{L}_S(\mathcal{H}_{\text{in}})$  describing how the state “looks” at early times, the “out” state  $S\Psi \in \mathcal{L}_S(\mathcal{H}_{\text{out}})$  describes how the state “looks” at late times. Hence, we have the following relation

$$|\psi\rangle_{\text{out}} = S |0\rangle_{\text{in}}. \quad (13)$$

This will tell us the spontaneous creation of particles by the gravitational field of black hole.

Using (7)-(12), with the orthogonality and completeness of the basis, one can obtain the relations between the operators of the “out” state and those of the “in” state

$$\begin{aligned}
S^{-1}a_{\text{out}}(\sigma)S &= \sum_{\sigma'} (a_{\text{in}}(\sigma')A_{\sigma'\sigma}^\dagger - a_{\text{in}}^\dagger(\sigma')B_{\sigma'\sigma}^\dagger), \\
S^{-1}a_{\text{out}}^\dagger(\sigma)S &= \sum_{\sigma'} (a_{\text{in}}^\dagger(\sigma')A_{\sigma\sigma'} - a_{\text{in}}(\sigma')B_{\sigma\sigma'}).
\end{aligned}
\tag{14}$$

The expected number of particles that spontaneously created in the mode  $\sigma$  can be expressed as,

$$\langle N(\sigma) \rangle =_{\text{out}} \langle \psi | a_{\text{out}}^\dagger(\sigma)a_{\text{out}}(\sigma) | \psi \rangle_{\text{out}}. \tag{15}$$

By means of (13) and (15), in addition with the Schwarzschild solution, the distribution of particle number can be given [16,21]

$$\begin{aligned}
\langle N(\sigma) \rangle &=_{\text{in}} \langle 0 | S^{-1}a_{\text{out}}^\dagger(\sigma)a_{\text{out}}(\sigma)S | 0 \rangle_{\text{in}} \\
&= (BB^\dagger)_{\sigma\sigma} \\
&= \frac{1}{e^{\omega_\sigma/T_H} - 1},
\end{aligned}
\tag{16}$$

with the form of standard thermal spectrum, which was first derived by S. Hawking, where  $T_H$  is the Hawking temperature

$$T_H = \frac{\kappa}{2\pi}, \tag{17}$$

with the surface gravity  $\kappa$  expressed as

$$\kappa = \frac{1}{4GM}, \tag{18}$$

where  $G$  is the Newton's gravity constant, and  $M$  is the ADM energy of black hole.

Above, the Hawking's theory of the evaporation of black hole was briefly reviewed. We considered a background spacetime of a Schwarzschild black hole from a collapsing spherical body, and only a single massless scalar field with a minimal coupling to gravity was taken into account for simplicity. The distribution of radiation particle-number was proven to be of the form of a perfect black-body spectrum according to the Hawking's theory. If the spacetime noncommutativity is taken into account, it should be expected that the radiation spectrum deviates from the standard thermal-form. We have proposed to describe the noncommutativity by means of the  $q$ -deformation scheme, *i.e.* we can mimic the particles live on the noncommutative spacetime near black hole by using the  $q$ -deformed particles (1), from which a different radiation spectrum can be derived. That means that we adopt an effective viewpoint that all the noncommutative effects are ascribed to the modification of the behavior of the radiation field meanwhile the background metric is preserved as the usual Schwarzschild metric. In the following, we will substitute the  $q$ -deformed particles to the normal ones, and re-calculate the black hole radiation spectrum in the framework of this toy model.

It has been proven that the basis of the Fock space would not be deformed by the  $q$ -deformation, namely the representation of the deformed algebra is the same as that of the normal algebra. That is to say, the relationship between the “out”-state and the “in”-state will be stood as (15) under the  $q$ -deformation. Hence, the expected number of  $q$ -particles in the  $\sigma$ -th mode of black hole radiation under the consideration of the spacetime noncommutativity will be

$$\begin{aligned}
\langle N(\sigma) \rangle_q &=_{\text{out}} \langle \psi | a_{\text{out}}^\dagger(\sigma)_q a_{\text{out}}(\sigma)_q | \psi \rangle_{\text{out}} \\
&=_{\text{in}} \langle 0 | S^{-1} a_{\text{out}}^\dagger(\sigma)_q a_{\text{out}}(\sigma)_q S | 0 \rangle_{\text{in}} \\
&=_{\text{in}} \langle 0 | S^{-1} [N_{\text{out}}(\sigma)]_q S | 0 \rangle_{\text{in}} \\
&=_{\text{in}} \langle 0 | S^{-1} \frac{q^{N_{\text{out}}(\sigma)} - q^{-N_{\text{out}}(\sigma)}}{q - q^{-1}} S | 0 \rangle_{\text{in}}
\end{aligned} \tag{19}$$

To evaluate this expectation value, an approximation is used [14]:

$$\langle N^k(\sigma) \rangle \simeq \langle N(\sigma) \rangle^k, \tag{20}$$

and further we get

$$\langle q^N \rangle \simeq q^{\langle N \rangle}. \tag{21}$$

Therefore, the approximate result can be obtained

$$\langle N(\sigma) \rangle_q \simeq \frac{q^{\langle N(\sigma) \rangle} - q^{-\langle N(\sigma) \rangle}}{q - q^{-1}} = [\langle N(\sigma) \rangle]_q. \tag{22}$$

For convenience, we re-express the deformation parameter as

$$\eta = \ln q, \tag{23}$$

Therefore, the expectation value of the number of radiated particles in the state  $\sigma$  of the deformed system can finally be of the form

$$\langle N(\sigma) \rangle_\eta = \frac{\sinh(\eta \langle N(\sigma) \rangle)}{\sinh \eta} = \frac{\sinh(\frac{\eta}{e^{\omega_\sigma/T_H} - 1})}{\sinh \eta}. \tag{24}$$

Next, let us calculate the energy spectrum. The quantum numbers of every mode can be labelled by  $\omega l m$ , namely  $\sigma = (\omega l m)$ . According to the particle number distribution (24), the number of particles with the angular momentum  $l$  per unit time in the frequency range  $\omega$  to  $\omega + d\omega$ , passing out through the surface of the sphere, can be got

$$\rho(\eta, \omega) = (2l + 1) \frac{\omega^2 d\omega}{2\pi^2} \frac{\sigma_l(\omega) \sinh(\frac{\eta}{e^{\omega/T_H} - 1})}{\sinh \eta}, \tag{25}$$

where  $\sigma_l(\omega)$  is the grey-body factor, and  $(2l + 1)$  is the degeneracy of the angular momentum. Thus the total outgoing energy flux (luminosity) of the black hole is given by

$$L_\eta = \frac{1}{2\pi^2 \sinh \eta} \sum_{l=0}^{\infty} (2l+1) \int_0^{\infty} d\omega \omega^3 \sigma_l(\omega) \sinh\left(\frac{\eta}{e^{\omega/T_H} - 1}\right). \quad (26)$$

This is just the total energy flux expression of the massless scalar radiation field with  $q$ -deformation that radiated by black hole under the consideration of spacetime noncommutativity. As we set  $\eta = 0$ , the original Hawking radiation will be recovered, namely

$$L_0 = \frac{1}{2\pi^2} \sum_{l=0}^{\infty} (2l+1) \int_0^{\infty} d\omega \omega^3 \sigma_l(\omega) \frac{1}{e^{\omega/T_H} - 1}. \quad (27)$$

The difference between (26) and (27) can be viewed as the consequence of the spacetime noncommutativity. Furthermore, it implies that some correlation effect among the radiation particles is introduced due to the noncommutativity. This point seems helpful for understanding the problem of information loss.

We now analyze the properties of the radiation energy-spectrum in detail. Since the  $s$ -wave in the energy spectrum is dominant, in addition that the gray-body factor of the  $s$ -wave is a constant, we will only analyze the  $s$ -wave for convenience. The radiation energy distribution of the  $s$ -wave can be expressed as

$$\rho_s(\eta, \omega) = A \frac{\omega^3}{2\pi^2 \sinh(\eta)} \sinh\left(\frac{\eta}{e^{\omega/T_H} - 1}\right), \quad (28)$$

where  $A$  is the  $s$ -wave gray-body factor, with the value event horizon area of black hole,  $A_H$ . If the parameter  $\eta$  is set to be 0, the distribution (28) will be reduced to the standard black-body radiation distribution. However, if the parameter  $\eta$  deviates from 0, even if slightly, the shape of the distribution function will be distorted evidently comparing to the shape of the black-body radiation, especially in the infrared region. Fig.1 shows the distributions as  $\eta$  is taken to be 0, 0.8, 1.4, and 2, respectively (from top to bottom, near the wave crests, in turn  $\eta = 0, 0.8, 1.4, 2$ ). We can see explicitly that the infrared divergency occurs when  $\eta$  deviates from 0. It implies that the physics in infrared region will be influenced by the ultraviolet modification that provided by the  $q$ -deformation. This phenomenon seems to take on some UV/IR correspondence principle. Moreover, it seems that some new physical mechanism hides in the infrared region. A natural treatment within the framework of our model is to introduce an infrared cut-off. It should be pointed out that the infrared cut-off always be necessary for the states count outside the black hole. In 't Hooft's brick wall model, an infrared cut-off was imposed in the WKB approximation for solving the number of states,  $g(E)$  [10]. For our prediction about the infrared cut-off, it looks more natural. In addition, we should mention that, by different considerations, Bekenstein and Mukhanov ever gave a minimal frequency  $\omega_0$ , as the fundamental quanta of the system of black hole, to characterize the quantum gravity of black hole [22]



$$\omega_0 = \frac{\alpha}{16\pi} \frac{1}{2GM}, \quad \alpha = 4 \ln 2. \quad (29)$$

Of course, there still exists another possibility, that is our method is unsuitable for the low frequency region. However, in any case, an infrared cut-off is necessary.

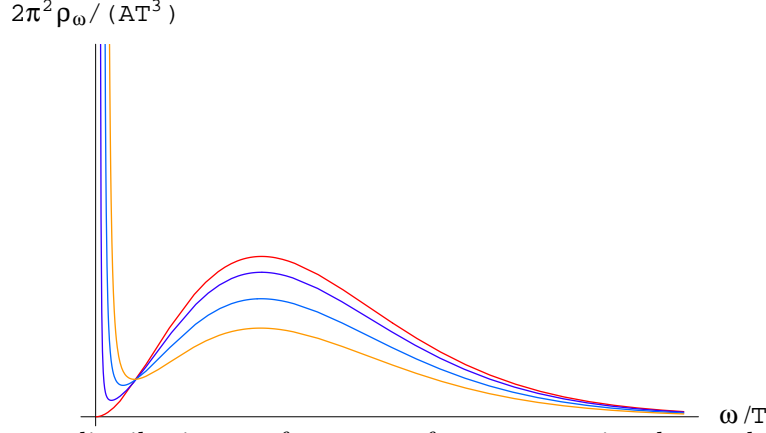


FIG. 1. The energy distributions vs frequency of  $s$ -wave as  $\eta$  is taken to be 0, 0.8, 1.4, and 2, respectively (from top to bottom, near the wave crests, in turn  $\eta = 0, 0.8, 1.4, 2$ ).

We now evaluate the position of the cut-off in frequencies. Since the exact solution is difficult to get, we only give an approximate evaluation. First, we have

$$\rho_s \sim \left( \frac{\eta^3}{6(-1+e^x)^3} + \frac{\eta}{-1+e^x} \right) x^3 \quad (30)$$

where  $x = \omega/T_H$ , and we have expanded the sinh-function up to cubic term. For solving the extremum, we have

$$\begin{aligned} 0 = \frac{\partial \rho_s}{\partial x} &\sim \frac{-\left(\eta \left(2(-1+e^x)^2 (3+e^x(-3+x)) + \eta^2 (1+e^x(-1+x))\right) x^2\right)}{2(-1+e^x)^4} \\ &\sim \frac{-(x(x^2 + 2(-2+x)x))}{2} \end{aligned} \quad (31)$$

In the second step,  $e^x \simeq 1+x$  is used. Then, the stationary points can be obtained

$$\omega_{m1} = T_H \frac{1}{2} (2 - \sqrt{2} \sqrt{2 - \eta^2}), \quad \omega_{m2} = T_H \frac{1}{2} (2 + \sqrt{2} \sqrt{2 - \eta^2}) \quad (32)$$

In our opinion, the minimal frequency of the system, if it exists, should be  $\omega_{m1}$ . In particular, for our interest, if we identify the  $\omega_{m1}$  with the Bekenstein's  $\omega_0$ , an appropriate value of  $\eta$  can be determined, that is

$$\eta \sim 1.35 \quad (33)$$

Naively, we can view this value as an important parameter to characterize the property of quantum gravity of black hole in some sense. Fig.2 shows the situation of  $\eta = 1.35$  contrasting to the standard case.

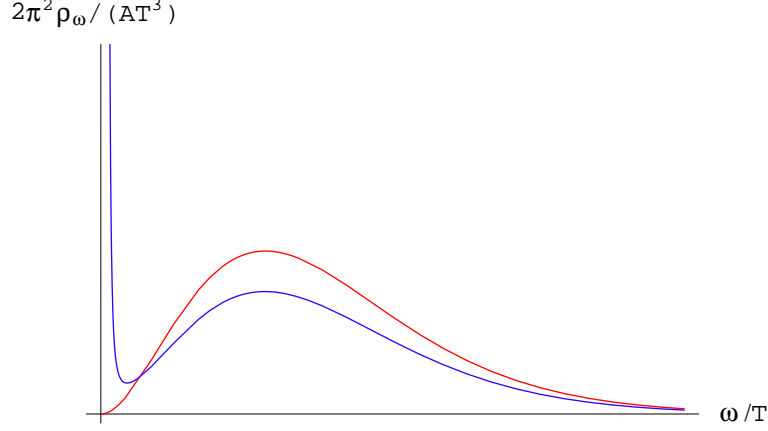


FIG. 2. The spectrum with  $\eta = 1.35$  comparing to the standard thermal spectrum.

Likewise, as an extension, for massless bosonic particles with spin  $s$ , the total flux can be also derived by the same  $q$ -deformation technique

$$L = \frac{g_s}{2\pi^2 \sinh \eta} \sum_{l=0}^{\infty} (2l+1) \int_0^{\infty} d\omega \omega^3 \sigma_{s,l}(\omega) \sinh\left(\frac{\eta}{e^{\omega/T_H} - 1}\right), \quad (34)$$

where  $g_s$  is used to denote the degeneracy of spin with the value 2 for nonzero spin and 1 for zero spin,  $\sigma_{s,l}(\omega)$  is the gray-body factor of black hole.

Furthermore, we switch to Schwarzschild black hole in  $D$  dimensions. The metric of such a black hole is

$$ds^2 = -\left(1 - \frac{r_H^{D-3}}{r^{D-3}}\right) dt^2 + \left(1 - \frac{r_H^{D-3}}{r^{D-3}}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2. \quad (35)$$

The relation between the horizon radius and the black hole mass is

$$r_H^{D-3} = \frac{16\pi G_D}{(D-2)\Omega_{D-2}} M, \quad (36)$$

where  $G_D$  is the Newton's gravity constant in  $D$  dimensions and  $M$  is the ADM energy of black hole. The black hole has a Hawking temperature

$$T_H = \frac{D-3}{4\pi r_H}. \quad (37)$$

The density of state of the radiation can be written as

$$g_s^{(D)} g_l^{(D)} \frac{\Omega_{D-2} \omega^{D-2} d\omega}{(2\pi)^{D-1}}, \quad (38)$$

where  $g_s^{(D)}$  and  $g_l^{(D)}$  are used to denote the degeneracies of spin and orbital angular momentum in  $D$  dimensional spacetime respectively. We also give the explicit expression of  $\Omega_{D-2}$ :

$$\Omega_{D-2} = \frac{2(\sqrt{\pi})^{D-1}}{\Gamma[\frac{D-1}{2}]}. \quad (39)$$

Then, the total flux of the spontaneously created  $q$ -deformed particles by the gravitational field of black hole in the  $D$  dimensional spacetime, can be obtained

$$L = \frac{\Omega_{D-2} g_s^{(D)}}{(2\pi)^{D-1} \sinh \eta} \sum_{l=0}^{\infty} g_l^{(D)} \int_0^{\infty} d\omega \omega^{D-1} \sigma_{s,l}(\omega) \sinh\left(\frac{\eta}{e^{\omega/T_H} - 1}\right). \quad (40)$$

In summary, a  $q$ -deformation prescription for introducing spacetime noncommutative effects into the black hole evaporation is proposed. We postulate a kind of  $q$ -deformed physical degrees of freedom to characterize the effects come from spacetime noncommutativity. The trick we use in this letter is that the spacetime noncommutative effects are ascribed to the modification of the behavior of the radiation field of black hole, and the calculation is performed under the usual Schwarzschild metric. Despite the suggestion of this toy model is highly speculative, it is still an attempt to probe the effects of noncommutative quantum gravity of black hole. A new spectrum of the black hole radiation is obtained by means of the  $q$ -deformation scheme, which deviates from the standard thermal spectrum evidently. It seems that some correlation is introduced to the radiation system. The existence of the infrared divergency implies that perhaps some unknown physical mechanism plays a role in the infrared region. In the framework of our model, an infrared cut-off is introduced. The fact that the change of the behavior of the infrared part originates from the ultraviolet modification embodies some UV/IR correspondence principle.

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